MORE THAN YOU EVER WANTED TO KNOW ABOUT SWITCH FROG NUMBERS AND SWITCH FROG ANGLES

By John Barrington

For the microbes, bugs and/or mice in the cars: THE LARGER THE FROG NUMBER, THE FASTER AND SMOOTHER IS THE RIDE THROUGH THE SWITCH.

1. Calculate the frog number ("N")

Visualize a line of symmetry through the center of the frog with the rails diverging on either side of that line. Pick a point x units down the line from the point of the frog and measure, perpendicular to the center line, left or right to the rail; call that measurement y units. The SPREAD is 2y; and N = x/2y.

Those of you who don't have the time to wade through the following detail sections can skip to the Summary.

I found a definition on the Web (http://my.inil.com/~dalemuir/definitions/definitionsFrog.html):
"One-half the cotangent of one-half the frog angle; or, the number of units of center line length in which the SPREAD is one unit".

2. Given the angle of divergence at the frog, calculate the frog number

Call the frog angle "alpha". The cotangent of alpha/2 is x/y (the adjacent side of the right triangle divided by the opposite side), and x/y = 2N:

cotan (alpha/2) = x/y = 2N
N = cotan (alpha/2) / 2.

Or using tangent = 1/cotangent:

tan (alpha/2) = y/x = 1/cotan (alpha/2)
N = cotan (alpha/2) / 2
N = (1 / tan (alpha/2)) / 2
N = 1/(2 tan (alpha/2))

(In case you don't have cotangent on your hand calculator, it is the inverse of the tangent--1/tan; tan is opposite over adjacent; cotan is adjacent over opposite. The cotan of alpha is also tan (90-alpha). If you have an ancient and honorable analog computer called a "slide rule", you probably have a cotangent "table" on it.)

So for an Atlas 17-degree switch angle
(17-degree frog angle):

- 1/2 the angle is 8.5 degrees,
- the tan of 8.5 degrees is 0.149
- the cotan of 8.5 is 1/tan = 6.66,
- so N = 6.66/2 = 3.33 (= too small!).

Wait! There's more! Actually, it turns out that the inverse of the sine of alpha
1/sin (alpha)
is very close to the frog number!
N = 1/sin(alpha)
For the Atlas 17-degree switch:
N = 1/sin(17) = 1/0.295 = 3.42 (= too small!).

3. Given the frog number, calculate the angle of divergence

From 1) above, N = x/2y so y/x = 1/2N.
The tangent of alpha/2 is also y/x. So
tan (alpha/2) = y/x = 1/2N,
alpha/2 = arctan (1/2N),
alpha = 2 arctan (1/2N).
As a check, solve 2) above for alpha:
N = 1/(2 tan (alpha/2)),
tan (alpha/2) = 1/2N,
alpha/2 = arctan (1/2N),
alpha = 2 arctan (1/2N).

Alternatively, for a close approximation, solve from 2) above:
N = 1/sin (alpha)
for alpha, getting:
alpha = arcsin (1/N)

4. Summary

Exact:
N = x/2y
N = 1/(2 tan (alpha/2))
alpha = 2 arctan (1/2N).

Close Enough:
N = 1/sin (alpha)
alpha = arcsin (1/N)

5. Table of Angles and Frog Numbers

ANGLE (deg.)	FROG #	COMMENT
5.72	10	Shinohara, ...
6	9.54	Peco Y
7.15	8	Atlas, ...
9.53	6	Shinohara, ...
10	5.72	Roco
11.4	5	...
12	4.76	Peco standard system
12.68	4.5	Atlas
14.25	4	...
15	3.80	Pilz/Railtech
17	3.33	Atlas Snap switch